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Approved on behalf of Managing Director, NPL, by Dr C Lea,  
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# **The Application of Finite Element Methods to the Design of Adhesive Joints**

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## **SUMMARY**

This article has been submitted for publication in the magazine Benchmark which is published by the association of finite element users NAFEMS. It summarises recent work in Project PAJ2 on the evaluation of the applicability of elastic-plastic models in FE packages for describing the deformation of tough adhesives at large strains. The limitations of these models is evident from the results of tests carried out in tension and shear on bulk specimens and in tension on butt-joint specimens of selected adhesives. The exponent Drucker-Prager model appears to satisfactorily describe the deformation behaviour of a toughened epoxy over the range of stress states generated by these tests. However, results on a toughened acrylic could not be accurately described using this model.

These observations are consistent with the interpretation that deformation under tension is controlled by a different plastic deformation mechanism from deformation under shear. If this is generally true for toughened adhesives, then the ability of the exponent Drucker-Prager model to describe deformation of the epoxy adhesive may be fortuitous.

A more full account of the data analysis and the measurement methods used to obtain the data presented will be given in a subsequent report from the Project.

## Introduction

A finite element analysis is a powerful tool in the design of adhesively bonded joints. An analysis is able to locate regions of stress and strain concentration in the adhesive, which will be sites for failure initiation, and to show how stress or strain levels can be reduced through changes to the size or geometry of the joint or the inclusion of adhesive fillets at the ends of the bond. In conjunction with a suitable criterion for failure of the adhesive, an analysis can be used to indicate when a joint will fail under different loading conditions such as impact, creep or fatigue. For this purpose, high accuracy is needed in the calculated stress and strain levels in the regions where failure initiates. The accuracy of the calculations depends on the validity of the materials model used by the analysis for describing the deformation behaviour of the adhesive. Since adhesives are generally tough materials, they can sustain large strains before failure and, under these conditions, relationships between stress and strain are highly non-linear.

The origin of non-linearity in a stress/strain curve for a polymeric adhesive can be explained in terms of enhanced creep brought about by an increase in molecular mobility caused by the application of a stress. At moderate stresses this non-linear deformation is recoverable. At higher stresses, the molecular mobility is high enough for yielding to occur by plastic flow which is not recoverable. Non-linear models developed specifically for polymeric materials have not been implemented in FE packages, so it is necessary to employ elastic-plastic models to describe deformation under large strain. Research at the National Physical Laboratory is currently evaluating the applicability and limitations of elastic-plastic models for use with adhesives. This article describes recent progress in this work.

## Materials Models

The most widely used of these models is based on the von Mises yield criterion. This can be expressed in the form

$$\sigma_T = \sqrt{3} J_{2D}^{1/2} \quad (1)$$

The parameter  $\sigma_T$  is a yield stress under uniaxial tension and  $J_{2D}$  is a function of the component of applied stress equal to the second invariant of the deviatoric stress tensor.

The initial value of  $\sigma_T$  defines the limit of linear behaviour and, in a strain hardening material,  $\sigma_T$  then increases with the plastic strain. It is known from tensile and shear or compression tests on adhesives that, in general, the yielding of these materials is sensitive to the hydrostatic component of applied stress in addition to the deviatoric component and cannot be accurately modelled by the von Mises criterion. A simple extension of the von Mises criterion that includes hydrostatic stress sensitivity takes the form

$$s_T = \frac{\sqrt{3}(I+1)}{2I} J_{2D}^{1/2} + \frac{(I-1)}{2I} J_1 \quad (2)$$

where  $\lambda$  is a material parameter representing the sensitivity of yield behaviour to the hydrostatic stress  $J_1$ . This criterion is implemented in certain FE materials models where it is referred to as the linear Drucker-Prager model.

The parameter  $\lambda$  is the ratio of yield stresses in uniaxial compression and uniaxial tension that have the same effective plastic strain. The experimental determination of  $\lambda$  requires the measurement of stress vs strain curves under two different states of stress. Uniaxial tension is an obvious choice for one of these stress states if representative bulk specimens are available. Shear is a convenient choice for the other stress state since test methods are available for obtaining shear data using bulk or joint test specimens.

### **Results for an Epoxy Adhesive**

Figure 1 shows true stress/true strain curves for a toughened, one-part epoxy supplied by Ciba Polymers under the code name LMD 1142. Analysis of these data using equation (2) leads to a value for the parameter  $\lambda$  of about 1.5 which is, to a good approximation, independent of plastic strain.

In order to assess the validity of this criterion over a wider range of stress states, predictions were made of the force/displacement curve for a butt-joint test specimen loaded in tension. The geometry of this joint is shown in figure 2. In this configuration, the adhesive experiences a predominantly triaxial tensile stress which is similar to the stress state present in adhesive joints in regions of high peel stress. Since failure of a joint will usually initiate in these regions, it is important to achieve accurate predictions of local stress and strain levels in these regions for confident predictions of joint performance.

Figure 2 shows experimental force/displacement data for a joint made with the epoxy between 25 mm diameter steel adherends and loaded in tension. The displacement was measured between reference points located on each adherend 3 mm from the adhesive layer. The predicted curve obtained by finite element analysis using the linear Drucker-Prager model in Abaqus is also shown. The input data were

- the tensile yield stress as a function of plastic strain, obtained from figure 1.
- the parameter  $\lambda = 1.5$  obtained from analysis of the data in figure 1.
- the flow parameter  $\psi = 25^\circ$ , obtained from measurements of the lateral contraction ratio in the tensile test that gave the tensile data in figure 1.

Comparison of this predicted curve with the experimental data demonstrates that the yield criterion in equation (2) is unable to describe deformation under the triaxial stress state existing in the butt-joint. (In fact the stress state in the above joint is very close to being pure hydrostatic tension when the load is high enough to cause extensive plastic deformation.)

The use of an alternative yield criterion was therefore explored given by

$$I \mathbf{s}_T^2 = 3J_{2D} + (I - 1)\mathbf{s}_T J_1 \quad (3)$$

This criterion is implemented in Abaqus in the exponent Drucker-Prager model. In a principal stress coordinate system, the surface represented by equation (3) for a particular value of  $\sigma_T$  is a paraboloid whose axis coincides with the hydrostatic stress axis. This criterion gives plastic deformation under hydrostatic tensile stress at lower stress levels than the equivalent conical surface given by equation (2) that passes through the same uniaxial tensile yield stress  $\sigma_T$ .

The measured force/deflection curve in figure 2 was used to obtain an approximate stress/strain curve for the butt-joint tensile test. These data, together with the tensile and shear data in figure 1, were then analysed using equation (3) to obtain a representative value for  $\lambda$ . Using the exponent Drucker-Prager model in Abaqus, a new force/deflection curve was predicted and is compared with experimental data in figure 2. Agreement is now satisfactory.

## Results for an Acrylic Adhesive

Results of tests on another adhesive however, indicate that the criterion in equation (3) is not generally valid. This adhesive is a toughened acrylic supplied by ITW Plexus under the code name MA310. Tensile and shear stress/strain curves obtained from bulk specimen tests are shown in figure 3. It can be seen that the hardening behaviour in tension is different from that in shear. Furthermore, initial results on butt-joint tensile tests indicate that the yield criterion in equation (3) cannot satisfactorily model the deformation behaviour of this adhesive under shear, tensile and hydrostatic tensile stress states. The reason for this is almost certainly linked to the occurrence of two distinct plastic deformation mechanisms in this material. It is known that for toughened plastics in general, plastic deformation in a tensile test is associated with a dilatational yield mechanism involving the formation of voids by cavitation or crazing. This mechanism would also control deformation under a hydrostatic tensile stress achieved with the butt-joint test. However, under a shear stress, void formation is inhibited, and deformation will be controlled by a shear yielding mechanism involving a constant, or nearly constant, volumetric strain.

A dilatational yield mechanism is also believed to contribute to plastic deformation under tension in the one-part epoxy shown in figure 1. This is indicated by a whitening of the specimen during tensile tests as well as a measured reduction in the lateral contraction ratio (Poisson's ratio) implying an increase in volume with increasing plastic strain. The apparent success with which equation (3) can be used to model the behaviour of this material over a range of stress states may be fortuitous and only possible because hardening under tension is similar to that under shear.

In order to realistically model toughened adhesives in a finite element analysis, a criterion for dilatational yielding will generally be required. The criteria in equations (1) or (2) are probably satisfactory for describing a predominantly shear yield mechanism. The materials model would then have to accommodate both of these criteria, and different hardening functions may need to be associated with each mechanism. The solver will need to select the appropriate yield criterion for the material in each element of the meshed geometry depending on the stress state within that element.

Current research at NPL is trying to identify a suitable form for the criterion for dilatational yielding. Further work is then required to establish for which adhesives and under which

situations this realistic modelling of deformation behaviour is needed and when simplifying approximations may be satisfactory.

## Acknowledgements

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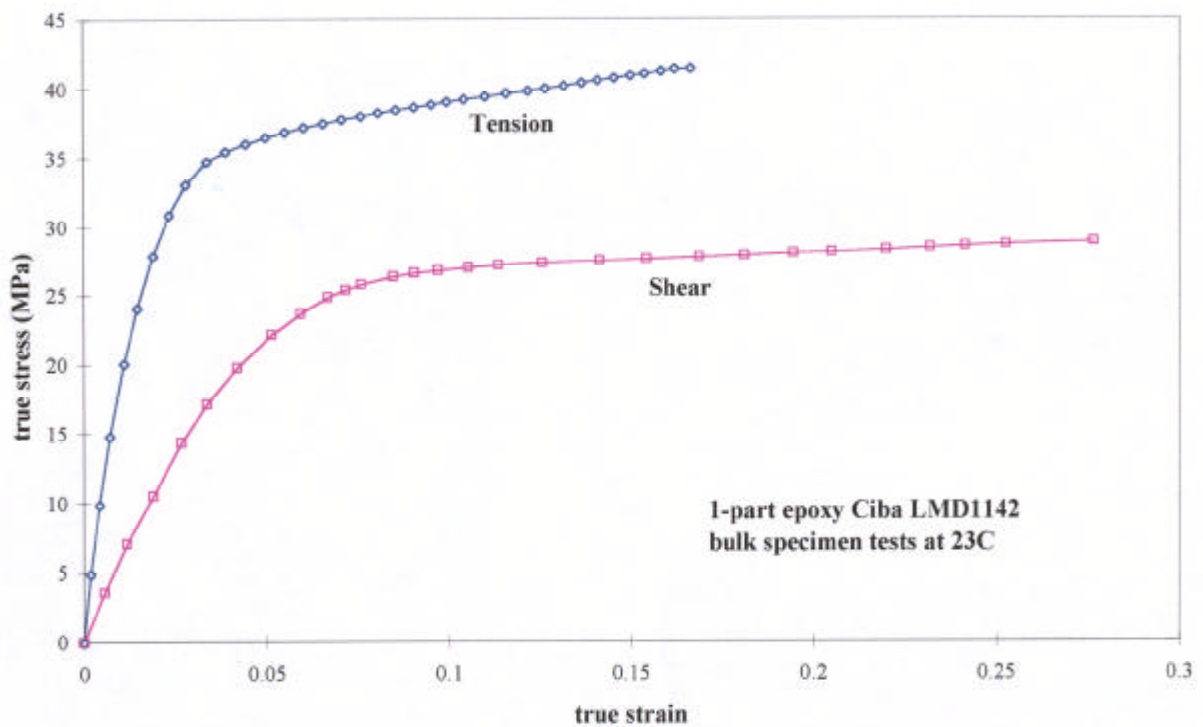


Figure 1. Stress/strain curves obtained under uniaxial tension and pure shear for a toughened epoxy adhesive.

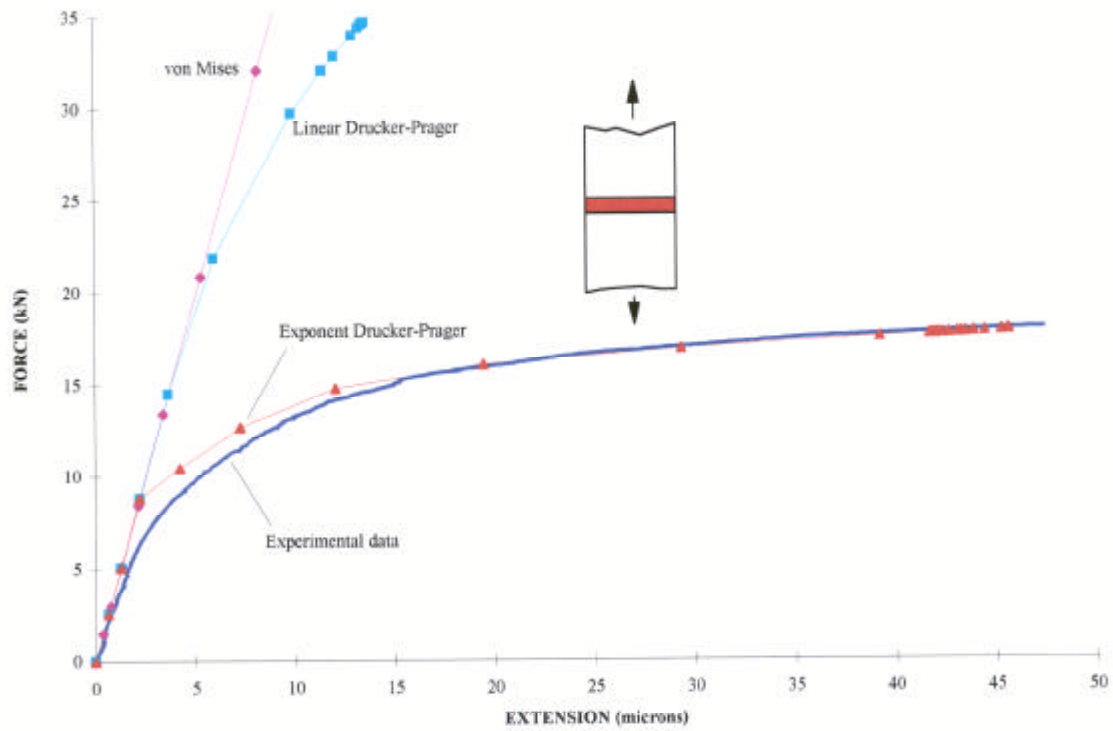


Figure 2: Comparison of measured and calculated force versus extension curves for a butt-joint loaded in tension. The extension is determined between points on each adherend located 3 mm from the bond. The bond thickness is 0.5 mm and the adherends are steel.

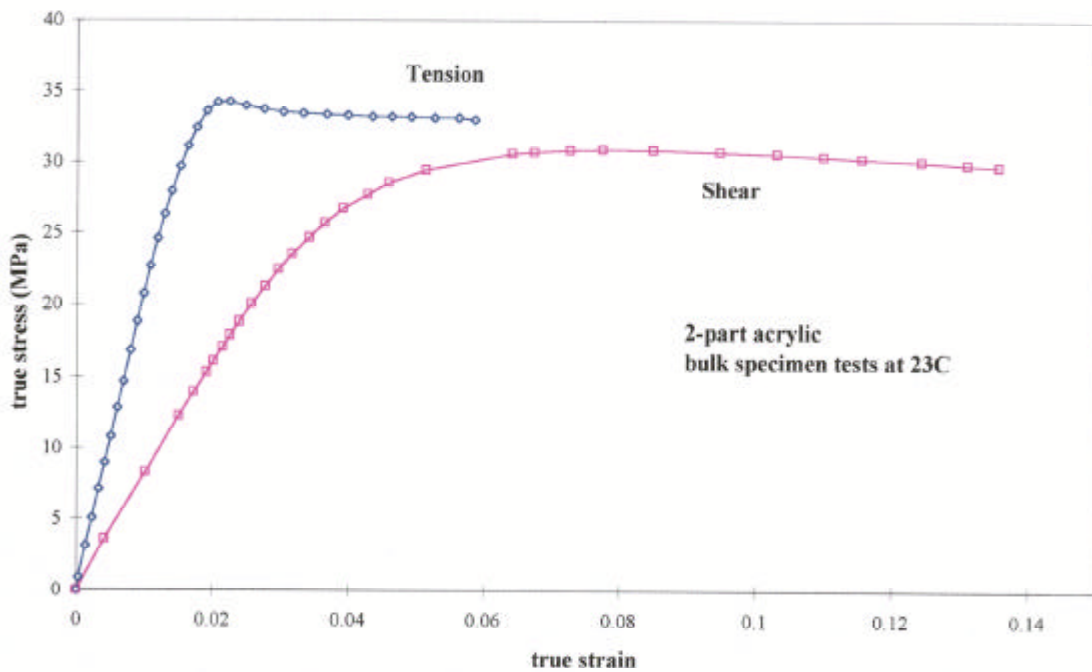


Figure 3. Stress/strain curves obtained under uniaxial tension and pure shear for a toughened acrylic adhesive.